PSD's and Fatigue

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Introduction

Fatigue damage is traditionally determined from time signals of loading, usually in the form of stress or strain. However, there are three design scenarios when a spectral form of loading is more appropriate. In this case the loading is defined in terms of its magnitude at different frequencies in the form of a Power Spectral Density (PSD) plot.

Firstly, the ***measurement engineer*** recording responses from in-service components or structures may be interested in PSD's because they are a very efficient way of defining a random stress or strain time history. By using a spectral description for the response of, say, an engine or suspension bracket, the requirement to collect and store large arrays of data can be avoided. In some situations the data might be collected and stored directly as PSD's, or alternatively a time series might be converted to a PSD using standard transformation techniques. Figure 1 highlights the equivalence between a time signal and its corresponding PSD. The transformation between ***time domain***, ie the time history of the loading, and the ***frequency domain***, ie a PSD, should not trouble the reader. The PSD simply shows the frequency content of the time signal and is an alternative way of specifying the time signal. It can be obtained from a time signal by utilizing the ***Fast Fourier Transform*** **(FFT).** Transforming from the frequency domain to the time domain is also a relatively easy task which can be done using the ***Inverse Fourier Transform*** **(IFT).** However, when transforming in this direction the random phase angles attributable to each frequency component have to be generated.

Secondly, the ***test engineer*** assessing the reliability of prototypes may be interested in spectral tools because such an approach allows the structural condition of the component to be monitored by continuous inspection of the system transfer function.

However, the most important benefit of working with PSD's is relevant to the ***structural analysis*** or designer because of the more sophisticated analysis options with which they can be used. In particular, dynamically responsive structures, or structures subjected to irregular loading patterns, can be efficiently and accurately analysed using a linear transfer function analysis to relate an input PSD of loading to an output PSD of response. Such approaches are widely used in nearly all engineering industries. The intercooler example shown in Figure 2 is one of many that could be used.

For all three of these design scenarios the fatigue designer is presented with a PSD of stress or strain with which to perform his fatigue calculation. There is therefore a requirement for a reliable, accurate and robust spectral fatigue design tool. Such a tool allows the designer to estimate the rainflow range content and hence fatigue damage from the PSD. This is the basis of the ***FATIMAS-SPECTRAL*** design tool which is currently being launched by nCode International Ltd.

Technical background

The first serious effort at providing a solution for estimating fatigue damage from PSD's was undertaken by SO Rice (1954). He developed the very important relationships for the number of upward zeros per second (E[0]) and peaks per second (E[P]) in a random signal expressed solely in terms of their spectral moments m_n. The irregularity factor, a very useful term for characterising the type of structural response encountered, was defined as E[0]/E[P]. This means that it varies between 0 (so called white noise containing a broad range of frequencies) and 1 (a classical narrow band signal containing only one predominant frequency).

$$E[0] \ ? \sqrt{\frac{m_2}{m_0}} \qquad E[P] \ ? \sqrt{\frac{m_4}{m_2}} \qquad ? \ ? \ \frac{E[0]}{E[P]} \ ? \ \sqrt{\frac{m_2^2}{m_0 m_4}}$$

The relevant spectral moments are easily computed from a one sided PSD G(f) in units of Hertz using the following expression.

$$m_n \ ? \ \overset{?}{\underset{0}{?}} f^{\,n} \ G(f) \ \ df$$

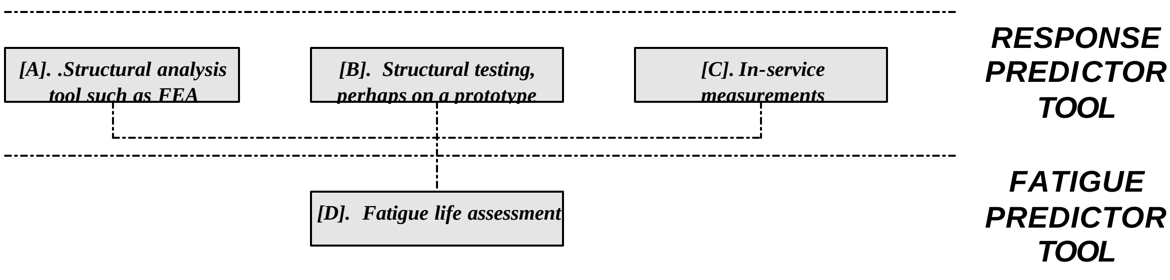
JS Bendat (1964) developed the theoretical basis for the so called ***Narrow Band*** solution. Again this expression was defined solely in terms of the spectral moments up to m_4. However, the fact that this solution was suitable only for a specific class of response conditions was an unhelpful limitation for the practical engineer.

Sherratt and Dirlik (Dirlik -1985) and then Sherratt and Bishop (1989, 1990) undertook much useful work in extending these restricted solutions during the period 1975 to 1990. This research work solved many of the theoretical problems which existed.

In particular a theoretical solution was obtained to estimate rainflow ranges of stress or strain from any class of PSD. However, many potential recipients of the technology were undoubtedly put off by the complexity of the techniques. Furthermore, implementation of the theoretical work into usable software was itself a complicated process with many potential numerical instability and inaccuracy pitfalls for the unsuspecting software writer. Because of this, work since 1990 has concentrated on making the techniques more widely applicable to the practical engineer (see for instance, Bishop - 1991, 1994, 1995). The rest of this article therefore addresses the application of this theoretical work using state of the art techniques. No theoretical background is provided since this is readily available in the references.

When should the techniques be used?

The fatigue designers task can be conveniently separated into ***response prediction*** and ***fatigue prediction*** as shown below. Both response and fatigue prediction will either be carried out in the time domain (ie, conventional stress or strain time series analysis) or in the frequency domain (ie using PSD's for the stress and strain information). The correct choice between time or frequency domain will be influenced by a number of issues which are primarily dependent on the type of analysis to be performed, ie, [A], [B] or [C] below. There are also a number of considerations which are relevant to the fatigue prediction stage [D]. These issues are discussed below.



[A]. Fatigue analysis with responses estimated using analytical or computational models such as Finite Element Analysis

An important question facing the designer is what are his analysis options? It may be that because of the nature of the structure being analysed the analysis route is already determined. For instance, many deepwater offshore oil platforms can only be satisfactorily designed in the frequency domain, thereby producing frequency domain results. In this case a frequency based fatigue calcula tion is the only option. Alternatively, the nature of the fatigue damage mechanism, or the structural system, may determine that only a time based approach is applicable. If either of these scenarios is true then the correct approach is already defined. However, there is often a choice and it is always worth considering if a frequency based analysis, perhaps in parallel with a time based approach, may provide an enhanced analysis capability. A recent analysis (Bishop et al, 1995) of an engine intercooler highlighted just such a situation where the analysis was considerably enhanced by the inclusion of a frequency based approach.

In general, for any system, there are three main analysis options as described below. Figure 2 highlights these options for the engine intercooler.

Deterministic (steady state). This is the most common type of analysis where the response of a structure to a deterministic, usually a sine wave, loading function is found. This type of analysis is severely limited for random or irregular loading functions because of its inability to properly characterise the true nature of the loading history.

Time domain (transient). In a world with infinitely powerful computers this would be the ideal form of analysis because the truly random nature of the loading can be retained and non-linearities can be catered for. Its main, and important, limitation is that because it is so computationally expensive it is not really suitable as a fast design tool.

Frequency domain (spectral). For systems which are not too non-linear (see next section) this approach offers all the advantages of the transient approach whilst being as fast as the deterministic method. In many design situations it is therefore an attractive option.

A very important question, if a spectral approach is being considered, concerns system linearity. In order to obtain the output (PSD) response of any system we need its ***transfer function***. This transfer function has to obey the following,

This is the basis of any frequency based structural analysis including FEA. Fortunately, even non-linear systems can often be linearised for certain operating conditions, thereby enabling a frequency based approach to be used.

[B]. Fatigue analysis with loads obtained from a prototype such as a test vehicle

If laboratory tests are being undertaken an important question is do the results need to be compared with, for instance, FEA results? The intercooler example demonstrates that frequency based FEA can be a powerful qualitative as well as quantitative tool for the reliability assessment of certain components. If this is likely then frequency based tests and fatigue calculations may be appropriate.

Another important question might be is any structural condition monitoring required, or beneficial? Frequency based measurements are very useful for determining any changes in structural behaviour or performance. Any change in the system transfer function is a useful indicator that some structural change has taken place, such as the growth of a crack. If such structural monitoring is likely to be beneficial then frequency based tests and fatigue calculations may be appropriate.

[C]. Fatigue analysis with loads measured directly from in-service components

If loads are being measured directly from in service components, are data storage problems likely to be an issue, either in terms of hard disk space or speed of acquisition? An acquisition rate of one fifth of that for time domain measurements can usually capture the frequency domain data required for a fatigue analysis with the same level of accuracy. Furthermore, there can also be a significant reduction in storage space required for the frequency domain data. For instance, a PSD can usually be characterised using approximately 1000 points. On the other hand, a truly random time signal may need 100,000 points or more for the same level of accuracy in the fatigue calculation.

[D]. Fatigue life assessment

An important and difficult question for the design engineer is concerned with whether the engineering process is stationary, Gaussian and random. Firstly, on the question of how Gaussian (sometimes referred to as 'normality') particular data is. If we calculate the percentage of time that response data spends within a particular stress bin and plot this as a ***probability density function*** **(pdf)** we require that its pdf follows the ***Gaussian*** bell shape. Fortunately there is a theoretical explanation to explain why nearly all engineering components and structures exhibit Gaussian behaviour. This is called the ***Central Limit Theorem***. This states, in very general terms, that the response of any system will be Gaussian as long as the number of processes contributing to this system response is reasonably large and that no one process dominates. This is true even if the individual processes are not Gaussian. Practical fatigue calculations have shown that the fatigue predictor tool used for this article is quite robust to some variation from a strictly Gaussian signal (see Bishop et al 1994).

If the signal is ***stationary*** it means that the general characteristics, such as rms, don't change with time. For most engineering processes this is true. Furthermore, even where the signal characteristics are changing slowly with time the complete response process can usually be broken up into a number of shorter stationary processes.

Finally, and perhaps most importantly, is the signal ***random***? If it isn't then a time based approach is likely to be the best approach. For instance, if a small number of transients dominate the fatigue damage then it is almost impossible for a frequency based fatigue predictor to properly identify these. This is an example, referred to in the Central Limit Theorem, where one process dominates the rest. If such transient or deterministic inputs are relatively small, in comparison to the rest of the response data, then a frequency based calculation may still be possible.

Irrespective of the above issues it is still worth considering if the advantages of working in the frequency domain outweigh any possible errors. If a frequency based fatigue predictor is coupled up with a frequency based load predictor, such as an FEA program, then a designer has the ability to undertake rapid design optimisation. He may then choose to use a time based calculation for the final 'proving' analysis calculation. In such a case any loss of accuracy involved in working in the frequency domain is far outweighed by the increased design capability. This is therefore something which should be carefully considered.

A simple worked example.

In order to illustrate the mechanism of spectral fatigue calculations it is worth performing some simple hand calculations on the PSD shown in Figure 3. Approximate hand calculations are performed in both the time and frequency domains. A ***FATIMAS-SPECTRAL*** calculation is then performed as a comparison.

Time domain. An approximate visualisation of the original time signal can be obtained by adding together two sine waves, one for each block in the PSD, where the amplitude of each is obtained (approximately) from 1.41 times the root mean square (rms) value. And since the rms of each block can be calculated (approximately) from its area we get the following stress ranges.

Sine wave 1 at 1Hz with a stress range of $\sqrt{10000}$ **x 1.41 x 2 = 282MPa**
Sine wave 2 at 10Hz with a stress range of $\sqrt{2500}$ **x 1.41 x 2 = 141MPa**

If we use a typical steel with S-N data of the form $N\ ?\ 1.0E\ ?\ 15 * S^{?4.2}$ we get

N(141MPa) = 9.4E+5 **N(282MPa) = 5.1E+4**
N(315MPa) = 3.2E+4 **N(423MPa) = 9.3E+3**

An approximate Palmgren-Miner damage calculation on the time signal then gives;

$$E[D]\ ?\ \frac{10}{9.4E\ ?\ 5}\ ?\ \frac{1}{9.3E\ ?\ 3} = \mathbf{1.18E\text{-}4}$$

This corresponds to a fatigue life of ***8462 secs***

Frequency domain by hand. Moments can be computed easily from the PSD using the expression given earlier.

$$m_n\ ?\ 40000 * 1^n\ *1\ ?\ 10000*10^n\ *1$$

m_0 **= 12 500** m_1 **= 35 000** m_2 **= 260 000** m_4 **= 25 010 000**

From which we can compute **E[0] = 4.6** upward zero crossings per second and
E[P] = 9.8 peaks per second
$?$ **= 0.465**
rms = $\sqrt{m_0}$ **= 112 MPa**
An equivalent sine wave magnitude = 112*1.41*2 = **315MPa**
E[D] = $\frac{9.8}{3.2E\ ?\ 4}$ **= 3.1E-4**

This corresponds to a fatigue life of ***3265 secs***

Note: This calculation is based on representing the constant frequency varying amplitude function with a sine wave.

Frequency domain calculation using FATIMAS-SPECTRAL.

Fatigue life using Narrow Band formula = ***1472 secs***
Fatigue life using Dirlik = ***7650 secs***

A more advanced example

The above calculation is useful for the practical designer as a means of identifying the steps involved in a frequency domain fatigue calculation. Since very close agreement between the various results would not be expected a more rigorous assessment of the techniques has been undertaken by applying the tools to a variety of engineering systems including, for example, automotive components, wind turbine blades, agricultural machinery and offshore platform joints. Results have shown that the tools are surprisingly accurate and robust for a wide range of applications. As an example a random time signal of length 5000 seconds has been analysed using ***FATIMAS-SPECTRAL.*** A representation of the time signal, and its PSD computed using standard FFT techniques, is given in Figure 5. The following results were obtained for a CLASSB weld specification.

Fatigue life estimate using rainflow counting etc................	**Time domain**	-	***20.48 Hours***
Fatigue life (frequency domain) using	**Dirlik solution**.	-	***18.07 Hours***
	Narrow Band solution	-	***10.58 Hours***
	Tunna solution	-	***59.10 Hours***
	Wirsching solution	-	***13.30 Hours***

Hancock solution	-	***16.26 Hours***
Kam and Dover solution	-	***20.16 Hours***
Steinberg solution	-	***9.91 Hours***

The choice of frequency domain solution will depend on the industrial application involved as well as the desired level of accuracy. Generally the Dirlik solution has been found to give the best results when compared with the corresponding time domain result. This is particularly true if the data is not truly Gaussian, stationary and random (see for instance Bishop et al - 1994). Further research work is ongoing at UCL, in partnership with nCode International, to look at mixed deterministic and random signals and more generally non-Gaussian signals. Any new advances which are made with the research will in due course, be incorporated into ***FATIMAS-SPECTRAL***.

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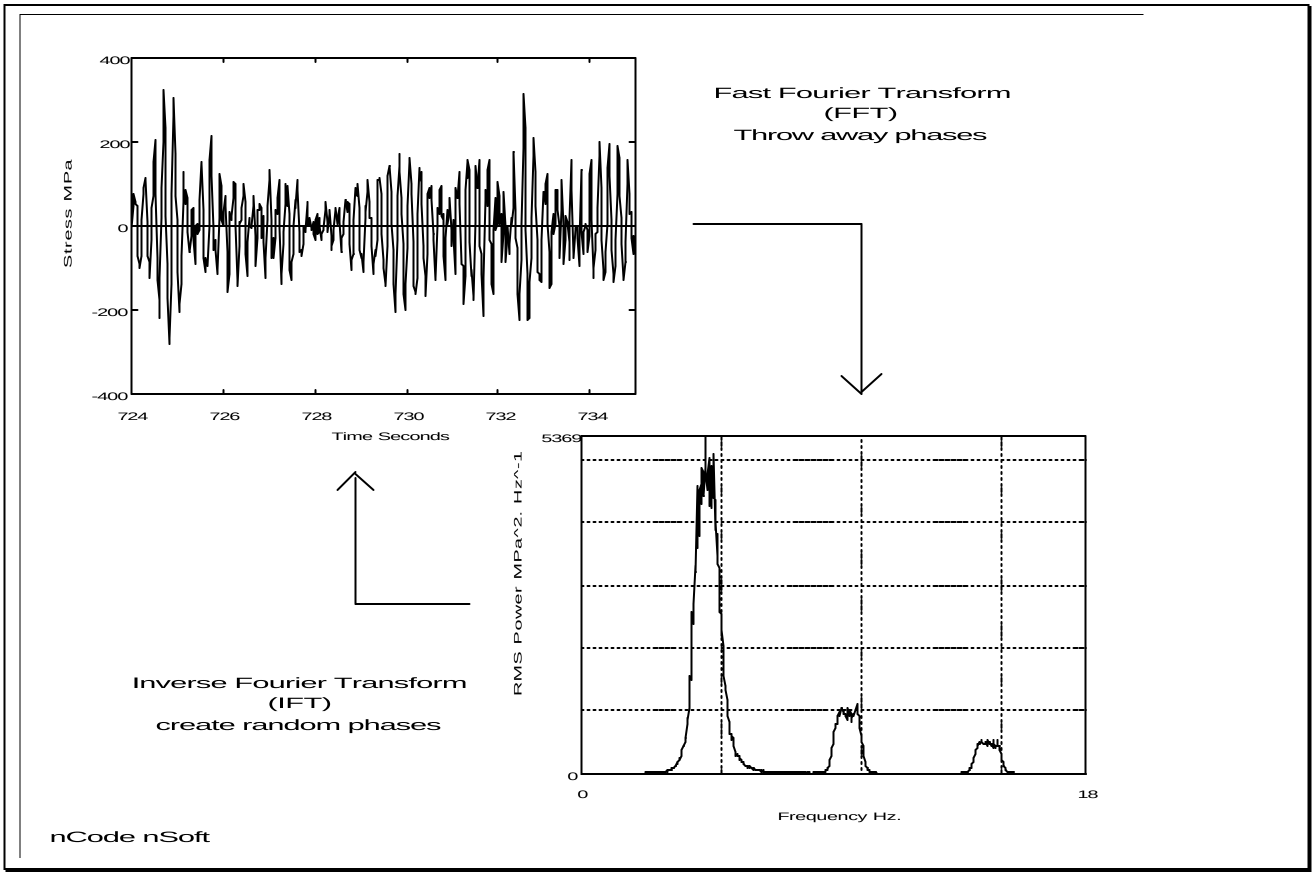

Figure 1. The relationship between a PSD and its equivalent time signal

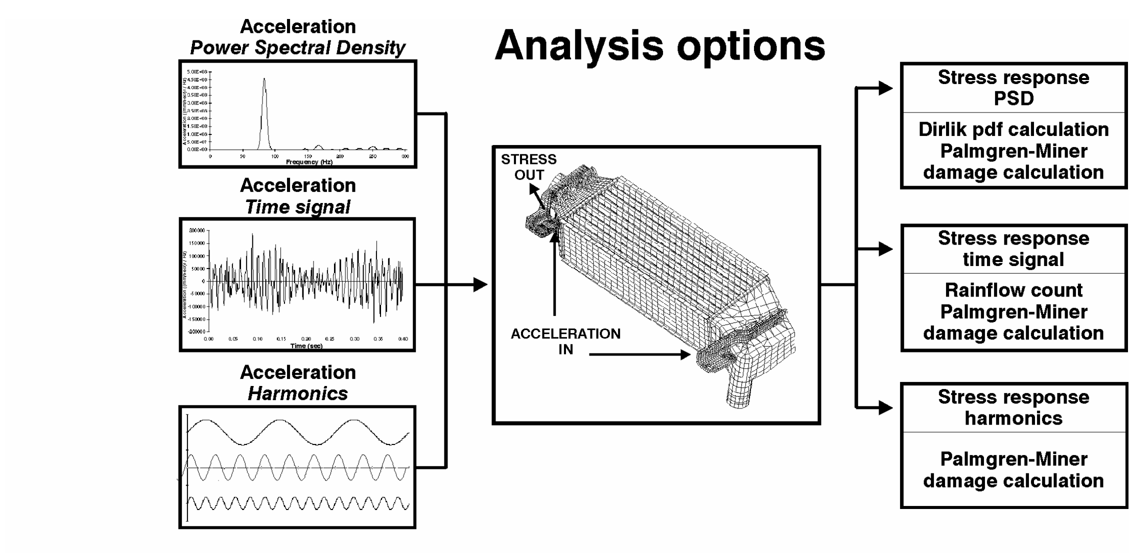

Figure 2. The analysis options available to the design engineer (see Bishop et al - 1995)

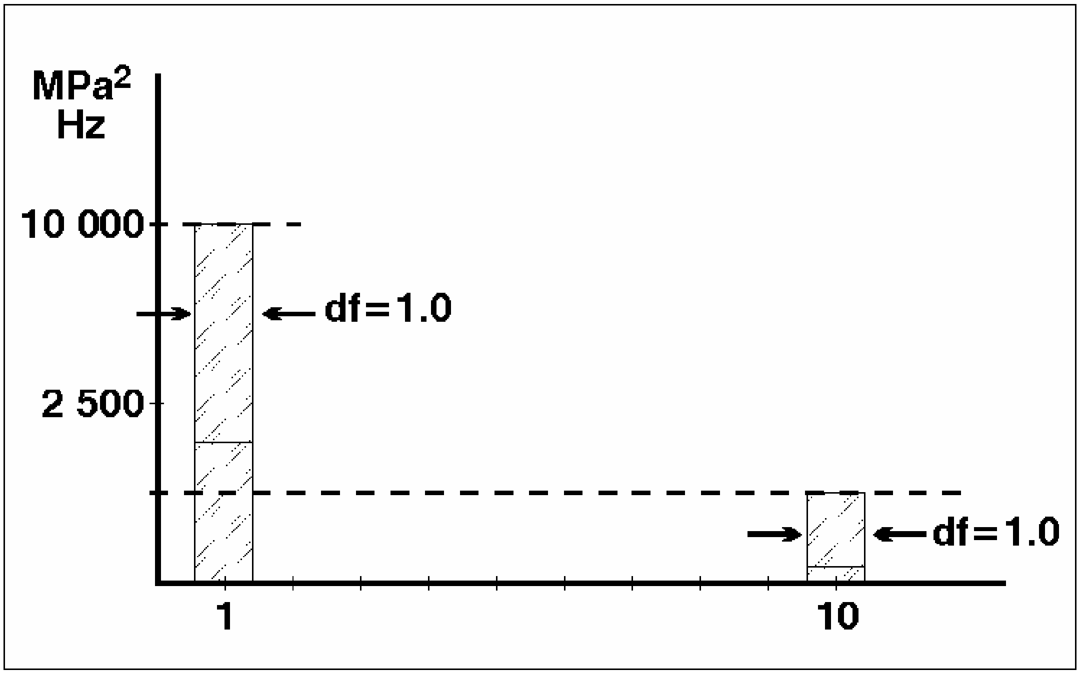

Figure 3. A simple two peaked PSD used to perform hand calculations

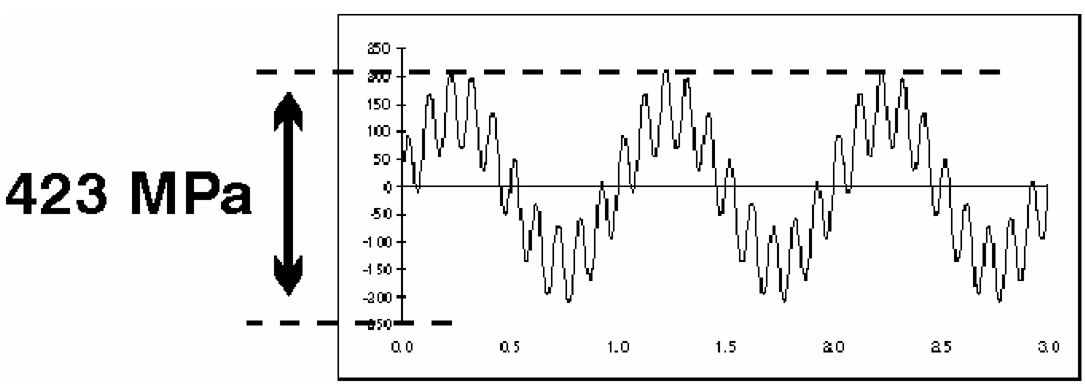

Figure 4. The sum of a sine wave at 1Hz with range 282MPa and sine wave at 10Hz with range 141MPa